

Key

2.3 Solve quadratics by factoring and factored form of a quadratic

Objectives:

Zero product property

Solve by factoring

Factored form

Example 1. Factor

a. $x^2+7x+12$

$$(x+3)(x+4)$$

b. $2x^2-5x-3$

$$(2x+1)(x-3)$$

c. x^2-9

$$(x+3)(x-3)$$

Definition:

Roots - solutions to a quadratic equation $0 = ax^2 + bx + c$

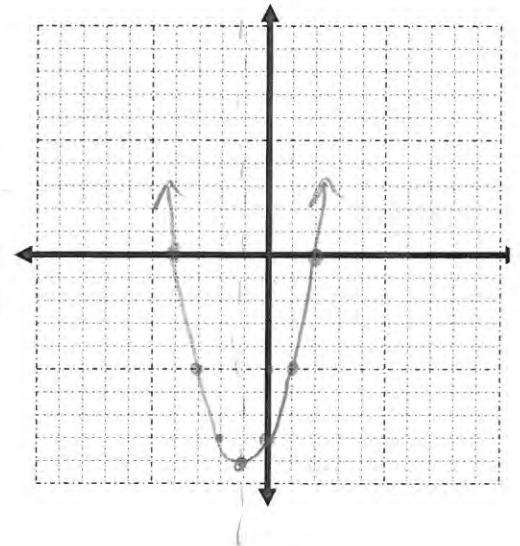
Zeros - where a function $f(x) = 0$ $f(x) = 0 = ax^2 + bx + c$

Example 2 Relate factors to zeroes

Graph the following $y = x^2 + 2x - 8$ $x = -b/2a = -2/2 = -1$

x	y
-1	-9
0	-8
1	-5
2	0

1 - 2 - 8
1 + 2 - 8
4 + 4 - 8
 $x = 2, -4$ are intercepts



Factor. $(x-2)(x+4)$

How do the factors relate to the zeroes (x intercepts)?

$$\left. \begin{aligned} (x-2)(x+4) &= 0 \\ x-2 &= 0 \quad x+4 = 0 \end{aligned} \right\}$$

Zero-Product Property If $a \cdot b = 0$ then $a = 0$ or $b = 0$

Words	If the product of two FACTORS is 0 then one or both of the factors is....
Algebra	If $AB = 0$ then $A = 0$ or $B = 0$ or BOTH = 0
Example	If $(x - 5)(x + 6) = 0$ then: $(x - 5) = 0$ or $(x + 6) = 0$ Meaning... $x = 5$ or $x = -6$

Solve $(x + 4)(x - 6) = 0$ $x + 4 = 0$ or $x - 6 = 0$
 $x = -4$ $x = 6$

4 steps for solving a polynomial equation or function

Zip it: Set the equation equal to 0. In a function replace $f(x)$ with 0.

Factor the equation.

Set each part equal to 0 and solve.

Check your answer on the calculator. Your answers should be the x intercepts if they exist.

Example 3

$x^2 + x = 42$

$x^2 + x - 42 = 0$
 $(x + 7)(x - 6) = 0$
 $x = -7$ $x = 6$

$2x^2 = -9x + 5$

$2x^2 + 9x - 5 = 0$
 $(2x - 1)(x + 5) = 0$
 $2x - 1 = 0$ $x + 5 = 0$
 $x = \frac{1}{2}$ $x = -5$

$2x^2 = 3x + 2$

$2x^2 - 3x - 2 = 0$
 $(2x + 1)(x - 2) = 0$
 $x = -\frac{1}{2}$ $x = 2$

Example 4. Find the zeroes of the quadratic function.

Marco hits a golf ball into the air from the third level of a driving range that is 48 ft high. The function $h(t) = -16t^2 + 32t + 48$ gives the height, h , in feet, of the golf ball t seconds after it is hit into the air. When will the ball hit the ground?

$0 = \frac{-16t^2 + 32t + 48}{-16}$ $\hookrightarrow h = 0$
 $0 = t^2 - 2t - 3$
 $0 = (t - 3)(t + 1)$
 $t = 3$ $t = -1$

Example 5. Determine positive and negative intervals.

Identify the intervals on which $y=x^2-2x-3$ is positive. Find intercepts

$$y = (x-3)(x+1)$$

$$x = -3 \quad x = -1$$



$$(-\infty, -1) \cup (3, \infty)$$

***Example 6.** Write a quadratic equation with roots $\frac{1}{2}$ and -5 .

$$x = \frac{1}{2}$$

$$x - \frac{1}{2} = 0$$

$$2x - 1 = 0$$

$$x = -5$$

$$x + 5 = 0$$

$$y = (2x-1)(x+5)$$

$$y = 2x^2 + 10x - x - 5$$

$$y = 2x^2 + 9x - 5$$

Example 6. Write an equation of a parabola with zeroes $(-2,0)$ and $(-1,0)$ that passes through the point $(-3, 20)$.

$$y = a(x-r)(x-r)$$

$$y = a(x+2)(x+1) \quad \text{Find } a$$

$$20 = a(-3+2)(-3+1)$$

$$20 = a(-1)(-2)$$

$$20 = 2a$$

$$10 = a$$

$$y = 10(x+2)(x+1)$$

Critical thinking. Find the value k that makes the statement true. -3 is a root of

$$2x^2 + kx - 21 = 0$$

$$2(-3)^2 + k(-3) - 21 = 0$$

$$18 - 3k - 21 = 0$$

$$-3k - 3 = 0$$

$$-3k = 3$$

$$k = -1$$

